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A Vector Error Correction Forecasting Model of the Greek Economy

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Abstract

This paper discusses the specification of Vector Error Correction forecasting models that are anchored by long-run equilibrium relationships suggested by economic theory. These relations are identified in, and are common to, a broad class of macroeconomic models. The models include variables such as the HICP, the unemployment rate, the real GDP, the GDP deflator, the 10-years government bond, the current account to GDP ratio and the exports to GDP ratio. The study examines the estimated model's stability, and following the "two-step approach", it assess the forecasting power of the estimated VECM by performing dynamic forecasts within and out of sample.

JEL Classification: C32, C51, C52, C53

Keywords: Cointegretion, Macroeconometric modelling, Vector Error Correction Model, Vector Autoregressive, forecast accuracy.

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Ένα Διανυσματικό Υπόδειγμα Διόρθωσης Λαθών για την Ελληνική Οικονομία

Περίληψη

Τα μοντέλα Διανυσματικής Αυτοπαλινδρόμησης αποτελούν ένα από τα πιο σημαντικά εργαλεία μακροοικονομικής και νομισματικής πολιτικής, τα οποία παρουσιάζουν τη διαχρονική δυναμική σχέση μεταξύ των διαφόρων μεταβλητών. Για την εκτίμηση της Διανυσματικής Αυτοπαλινδρόμησης χρησιμοποιούμε στοιχεία που καλύπτουν την περίοδο από το πρώτο τρίμηνο του 2000 μέχρι και το δεύτερο τρίμηνο του 2017. Κύρια πηγή των δεδομένων είναι η Ελληνική Στατιστική Αρχή και ο ΟΟΣΑ. Κάποιες μεταβλητές που ήταν διαθέσιμες σε μηνιαία συχνότητα μετατράπηκαν σε τριμηνιαία λαμβάνοντας το μέσο όρο του τριμήνου. Στη συνέχεια πραγματοποιήθηκε εποχική διόρθωση των μεταβλητών, για όσες μεταβλητές δεν ήταν εποχικά διορθωμένες, με χρήση του φίλτρου TRAMO/SEATS. Το σύνολο των δεδομένων περιλαμβάνει το πραγματικό ΑΕΠ, το ποσοστό ανεργίας, τον εναρμονισμένο δείκτη τιμών καταναλωτή, το ισοζύγιο τρεχουσών συναλλαγών ως προς το ΑΕΠ, το λόγο των εξαγωγών ως προς το ΑΕΠ, τον αποπληθωριστή του ΑΕΠ, την απόδοση του 10ετούς ομολόγου του δημοσίου, τις τιμές πετρελαίου και το πραγματικό ΑΕΠ της Ευρωζώνης.

Συγκεκριμένα, εκτιμώνται δύο υποδείγματα. Στο πρώτο υπόδειγμα χρησιμοποιούνται τέσσερις ενδογενείς μεταβλητές (πραγματικό ΑΕΠ, ποσοστό ανεργίας, εναρμονισμένος δείκτης τιμών καταναλωτή και ισοζύγιο τρεχουσών συναλλαγών ως προς το ΑΕΠ) και δύο εξωγενείς (τιμές πετρελαίου και πραγματικό ΑΕΠ της Ευρωζώνης). Το δεύτερο υπόδειγμα περιλαμβάνει πέντε ενδογενείς μεταβλητές και συγκεκριμένα το πραγματικό ΑΕΠ, το ποσοστό ανεργίας, τον αποπληθωριστή του ΑΕΠ, την απόδοση του 10ετούς ομολόγου του δημοσίου και το λόγο των εξαγωγών ως προς το ΑΕΠ.

Ο σωστός προσδιορισμός των υποδειγμάτων στηρίζεται στην έννοια της στατιστικής επάρκειας, σύμφωνα με την οποία τα στατιστικά στοιχεία υποστηρίζουν τις υποθέσεις του προϋποτεθειμένου μοντέλου Διανυσματικής Αυτοπαλινδρόμησης. Η στατιστική επάρκεια ενός μοντέλου είναι ζωτικής σημασίας για την εγκυρότητα των αποτελεσμάτων γιατί, αν τα στοιχεία που εξετάζονται δεν υποστηρίζουν τις υποθέσεις του μοντέλου που χρησιμοποιείται, τότε οποιαδήποτε συμπεράσματα πιθανώς να είναι παραπλανητικά. Ακολουθώντας τη μοντέρνα βιβλιογραφία στην οικονομετρία, χρησιμοποιούνται μια σειρά από ελέγχους στατιστικής επάρκειας για κανονικότητα, γραμμικότητα, ομοσκεδαστικότητα, στασιμότητα και μη γραμμική συσχέτιση. Ως εκ τούτου, τα υποδείγματα αποτελούνται από χρονικές

υστερήσεις και χρονική τάση. Παράλληλα, μέσα στα πλαίσια των συγκεκριμένων υποδειγμάτων διεξάγεται ανάλυση Granger Causality, η οποία επιβεβαιώνει τη βραχυχρόνια σχέση που συνδέει τις μεταβλητές.

Τα υποδείγματα Διανυσματικής Αυτοπαλινδρόμησης μας περιγράφουν μόνο τη βραχυχρόνια σχέση των μεταβλητών. Η μακροχρόνια συμπεριφορά των μεταβλητών δίνεται από τα διανυσματικά υποδείγματα διόρθωσης λαθών (Vector Error Correction Model-VECM), τα οποία έχουν ως απαραίτητη προϋπόθεση την ύπαρξη συνολοκλήρωσης. Έτσι, γίνεται έλεγχος για την ύπαρξη συνολοκλήρωσης ακολουθώντας τη μεθοδολογία των Johansen και Juselius (1990,1992). Τα αποτελέσματα δείχνουν ότι στα υποδείγματα υπάρχουν σχέσεις συνολοκλήρωσης, οι οποίες επιτρέπουν την εκτίμηση του υποδείγματος διόρθωσης λαθών. Παράλληλα, μέσα στα πλαίσια του διανυσματικού υποδείγματος διόρθωσης λαθών πραγματοποιείται και ανάλυση διακύμανσης (Variance Decomposition).

Το βασικότερο στοιχείο που χαρακτηρίζει τα διανυσματικά υποδείγματα διόρθωσης λαθών είναι η προβλεπτική τους ικανότητα. Ακολουθώντας τη βιβλιογραφία, γίνεται εκτίμηση του κάθε υποδείγματος και στη συνέχεια αναλύεται η προβλεπτική του ικανότητα διεξάγοντας δυναμική πρόβλεψη εντός (2000:1-2014:4) και εκτός (2015:1-2016:4) δείγματος. Τα αποτελέσματα δείχνουν ότι η προβλεπτική ικανότητα των μοντέλων είναι πολύ καλή, και συνεπώς τα μοντέλα αυτά αποτελούν αναμφισβήτητα πολύ χρήσιμα εργαλεία.

1. Introduction

Numerous studies of macroeconomic time-series data suggest a need for careful specification of the model's multivariate stochastic structure. Following the classic work of Nelson and Plosser (1982), many studies have demonstrated that macroeconomic time series data likely include components generated by permanent (or at least highly persistent) shocks. Yet, economic theory suggests that at least some subsets of economic variables do not drift through time independently of each other; ultimately, some combination of the variables in these subsets, perhaps nonlinear, reverts to the mean of a stable stochastic process. Granger (1981) defined variables whose individual data generating processes are well-described as being driven by permanent shocks as integrated of order 1, or I(1), and defined those subsets of variables for which there exist combinations (linear or nonlinear) that are well described as being driven by a data generating process subject to only transitory shocks as cointegrated.

Many cointegration studies have shown that some individually I(1) variables—including real money balances, real income, inflation, and nominal interest rates—may be combined in linear relationships that are stationary, or I(0). Evidence on the stationarity of linear money demand relations has been presented by Hoffman and Rasche (1991), Johansen and Juselius (1990), Baba, Hendry, and Starr (1992), Stock and Watson (1993), Hoffman and Rasche (1996a), Crowder, Hoffman and Rasche (1999) and Lucas (1994), among others. Evidence in favor of an equation that links the income velocity of money to nominal interest rates, in several countries, is presented by Hoffman, Rasche and Tieslau (1995). Mishkin (1992), Crowder and Hoffman (1996) and Crowder, Hoffman and Rasche (1999) present evidence of a Fisher equation, and Campbell and Shiller (1987, 1988) have examined cointegration among yields on assets with different terms to maturity.

Anderson, Hoffman and Rasche (2002) estimate a VECM model for the US that includes six variables – real GDP, the GDP deflator, the CPI, M1, the federal funds rate, and the constant-maturity yield on 10-year Treasury securities – and four cointegrating vectors. Their forecasts from the model for the 1990s compare favorably to alternatives, including those made by government agencies and private forecasters. Christofidis, Kourtellos and Stylianou (2004) estimate a four variable VAR as well as a VECM model for the Cyprus economy using nominal gross domestic product, total liquidity (M2), the average deposit rate, and the consumer price index. The VECM estimation is extremely significant, since it not only provides useful information on the long run equilibrium relationship of the variables but, in addition, is the basis for forecasting analysis.

Our study describes an application of VECM models to the forecasting of important Greek macroeconomic variables in the following quarters. We use quarterly data for the HICP, the unemployment rate, the real GDP, the GDP deflator, the current account to GDP ratio, the exports to GDP ratio and the 10-years government bond. An out-of-sample assessment shows that the quality of the forecasts supplied by this model is satisfactory.

Our paper is organized as follows. Section 2 describes the VECM models as well as the associated estimation and forecasting methods. Section 3 presents the data used in our study and examines the forecasting performance of VECM models tested on their sample base and on an out-of-sample basis.

2. Vector Autoregressive models and Cointegration Analysis

2.1. Vector Autoregressive models

The Vector Autoregressive model (VAR) was popularized by Sims (1980) as a model which disregards the theoretical restrictions of simultaneous equation, or structural, models. The model is formed by using characteristics of our data; therefore there are no restrictions that are based on economic theory. However, economic theory still has an importance for VAR modeling when it comes to the selection of variables. According to Sims there should not be any distinction between endogenous and exogenous variables when there is true simultaneity among a set of variables. The VAR model can be seen as a generalization of the univariate autoregressive model and is used to capture the linear interdependencies in multiple time series. Its purpose is to describe the evolution of a set of *k* endogenous variables based on their own lags and the lags of the other variables in the model.

Regarding the assumptions of the VAR model, there are not many that need to be considered. This is because the VAR model lets the data determine the model and uses no or little theoretical information about the relationships between the variables. Except for the assumption of white noise disturbance terms, it is beneficial to assume that all the variables in the VAR model are stationary, to avoid spurious relationships and other undesirable effects. If the variables are not stationary, they have to be transformed into stationarity by taking differences. A standard *k* variables VAR model of order *p* has the following form:

$$y_t = \beta_0 + \sum_{i=1}^{p} A_i y_{t-i} + BX_t + u_t$$

where $y_t \in R^k$ is the $k \times 1$ vector of the I(1) endogenous variables. X is a vector of deterministic variables which might include a trend and dummies, $\beta_0 \in R^k$ is a vector of intercepts, A_i is a $k \times k$ coefficient matrix, B is a coefficient matrix, and $u_t \in R^k$ is a vector of innovations.

The selection of the final VAR for every combination of variables is based on the criterion of statistical adequacy. A model is said to be statistically adequate if all the underlying assumptions of the model are supported by the data. This is crucial because, if our model is statistically adequate, we are able to support statistically hypothesis testing, forecasting, causality tests, etc. More precisely, we may test for normality, for static and dynamic heteroskedasticity, for serial correlation, for non linearity, for omitted variables, as well as stability. An important issue in model specification is also model parameter stability. Often structural breaks characterize macroeconomic variables over a long period of time.

2.2. Cointegration Analysis and Vector Error Correction Model

Economic theory often suggests that certain groups of economic variables should be linked by a long-run equilibrium relationship. Although the variables may drift away from equilibrium for a while, economic forces may be expected to act so as to restore equilibrium. Variables which are I(1) tend to diverge as $n \rightarrow \infty$ because their unconditional variances are proportional to the sample size. Thus it might seem that such variables could never be expected to obey any sort of long-run equilibrium relationship. But, in fact, it is possible for a group of variables to be I(1) and yet for certain linear combinations of those variables to be I(0). If that is the case, the variables are said to be cointegrated. If a group of variables is cointegrated, they must obey an equilibrium relationship in the long run, although they may diverge substantially from equilibrium in the short run.

A vector error correction model (VECM) is a restricted VAR model in differences. The VECM specification restricts the long-run behavior of the endogenous variables to converge to their long-run equilibrium relationships, while allowing for short-run dynamics (see, for example, Engle and Granger (1987). This is done by including an error correction mechanism (ECM) in the model, which has proven to be very useful when it comes to modeling non-stationary time series. The VECM formulation of the corresponding VAR representation can be written as:

$$\Delta y_t = \beta_0 + \sum_{i=1}^{p-1} \Gamma_i y_{t-i} + \Pi y_{t-1} + BX_t + u_t$$

The Πy_{t-1} is the error correction term and the $k \times r$ matrix *Π* shows how the system reacts to deviations from the long-run equilibrium. The short-run dynamics are ruled by Γ_i. When *r* is zero then a process in differences is appropriate and when $r = k$ then in levels. For $0 < r < k$ there exists an ECM that pushes back deviations from the long-run equilibrium (characterized by the co-integrating relations). For a solid review of the VECM, see, for example, Johansen (1988, 1991, 1995).

We may test for cointegration in the context of a system of equations. Johansen and Juselius (1990, 1992) propose a test of this type, which is based on canonical correlations, using a Likelihood Ratio Test. The application of this test requires the inclusion of exogenous variables, e.g., an intercept and trend in the long-run relationship and a linear trend in the short-run relationship. In addition, Johansen, Mosconi and Nielsen (2000) as well as Hungnes (2005) consider the presence of dummies in the cointegration relationship when the variables are affected by a number of breaks.

After finding evidence supporting the existence of a cointegrating relationship among the examined variables, someone may estimate a VECM. As mentioned before, a VEC Model is a restricted VAR which has cointegration relations built into the specification so that it restricts the long-run behaviour of the endogenous variables to converge to their cointegrating relationships while allowing for short-run adjustment dynamics. The cointegration term is known as the correction term since the deviation from long-run equilibrium is corrected gradually through a series of partial short-run adjustments.

In the context of the VECM estimation, Pairwise Granger Causality Tests and Impulse Response Function analysis can be used for economic policy evaluation (see, e.g. Sims, 1980). The Impulse Response Function is the path followed by y_t as it returns to equilibrium when we shock the system by changing one of the innovations (u_t) for one period and then returning it to zero.

Another way of characterizing the dynamic behaviour of a VAR system is through Forecast Error Variance Decomposition, which separates the variation in an endogenous variable into the component shocks to the VAR. If, for example, shocks to one variable fail to explain the forecast error variances of another variable (at all horizons), the second variable is said to be exogenous with respect to the first one. The other extreme case is if the shocks to one variable explain all forecast variance of the second variable at all horizons, so that the second variable is entirely endogenous with respect to the first.

Since cointegration is present, it is extremely significant to model the short-run adjustement structure, i.e the feedbacks to deviations from the long run relations, because it can reveal information on the underlying economic structure. Modeling the feedback mechanisms in cointegrated VAR models is typically done by testing the significance of the feedback coefficients. These tests are called weak exogeneity tests, because certain sets of zero restrictions imply long run weak exogeneity with respect to the cointegrating parameters. The concept of weak exogeneity was defined by Engle, Hendry and Richard (1983) and is closely related to testing the feedback coefficients. If all but one variable in a system are weakly exogenous, then efficient inference about the cointegration parameters can be conducted in a single equation framework. Choosing valid weak exogeneity restrictions is of major importance, because policy implications are sometimes based on the short-run adjustment structure. According to Johansen (1995), there is a Likelihood Ratio Test that may be used to test weak exogeneity.

The VECM presents not only the long-run relationship of the variables, but it has an additional significant advantage: forecasting. According to Anderson, Hoffman and Rasche (2002) we may perform a “two-stage technique”, where we estimate an economic relation using the technique of a VECM and, on a second stage, we assess the quality of forecast outcome. Thus, in the context of stochastic simulation analysis we apply dynamic forecasts (multi-step forecasts) using a large number of iterations within and out of the time bounds of the observations of the sample. After forecasting, we assess how far the estimated model has approximated the real-historical values. The closer the forecasts are to the real values, the better the forecasting power of the VECM considered. The algorithm used for the implementation of iterations is the well-known Gauss-Seidel, which works by evaluating each equation in the order that it appears in the model, and uses the new value of the left-hand variable in an equation as the value of that variable when it appears in any later equation.

3. Empirical analysis

3.1. Data

Our data set covers the period from the first quarter of 2000 until the second quarter of 2017. All series were downloaded from Eurostat and OECD databases. Some variables that published monthly have been converted to quarterly frequency by taking the average of the corresponding quarter. Our data set includes the real GDP, the unemployment rate, the harmonized index of consumer prices, the current account to GDP ratio, the exports to GDP ratio, the GDP deflator, the 10-years government bond, the oil price and the real GDP of euro area.

All the series, except for the harmonized index of consumer prices, the current account to GDP ratio and the oil price, were seasonally adjusted. So, using the TRAMO/SEATS filter we proceed to seasonal adjustment of these series. Table 1 presents briefly the descriptive statistics for those variables, while Figure 1, Figure 2 and Figure 3 presents the level, the level in logarithms and the first difference graph respectively.

Table 1: Descriptive Statistics

	Mean	Median	Maximum	Minimum	Std. Dev.
Real GDP	52.912,24	51.941,83	63.333,13	45.479,00	6.076,97
Real GDP EURO	2.350.879,00	2.391.911,00	2.570.921,00	2.099.097,00	120.694,10
Unemployment rate (%)	15,21	10,78	27,83	7,53	7,17
HICP	90,89	94,08	103,74	70,12	10,86
Deflator	91,29	94,94	101,82	74,30	8,47
Oil Prices	64,56	58,62	122,46	19,35	31,84
GB10Y (%)	7,60	5,60	25,40	3,41	4,93
Current Account to GDP (%)	-0,08	-0,08	0,01	-0,16	0,05
Exports to GDP (%)	24,60	23,25	34,33	18,33	4,76

Figures 1 and 2 suggest that most series have a trend, whereas the presence of structural breaks is also obvious. It is crucial to incorporate the structural breaks using dummies in the VAR model, since they affect their short run as well their long-run relationship. At first glance, it seems that the real GDP, the unemployment rate, the real GDP of euro area, the ten year government bond and the oil price have a structural break in 2008. The harmonized index of consumer prices and the current account to GDP ratio have a structural break in 2010. The influence of the structural break is more obvious in Figure 3, where the series are presented in first differences.

Figure 1: level presentation of the variables

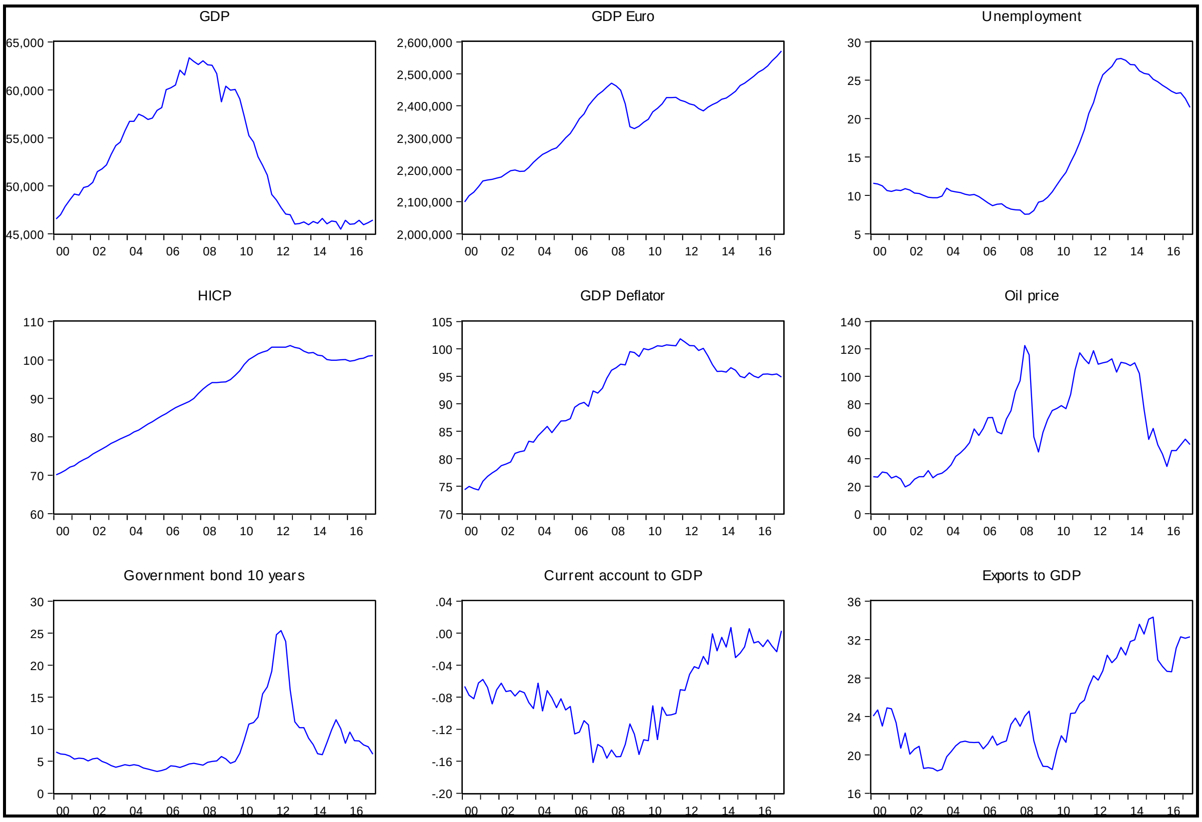


Figure 2: log presentation of the variables

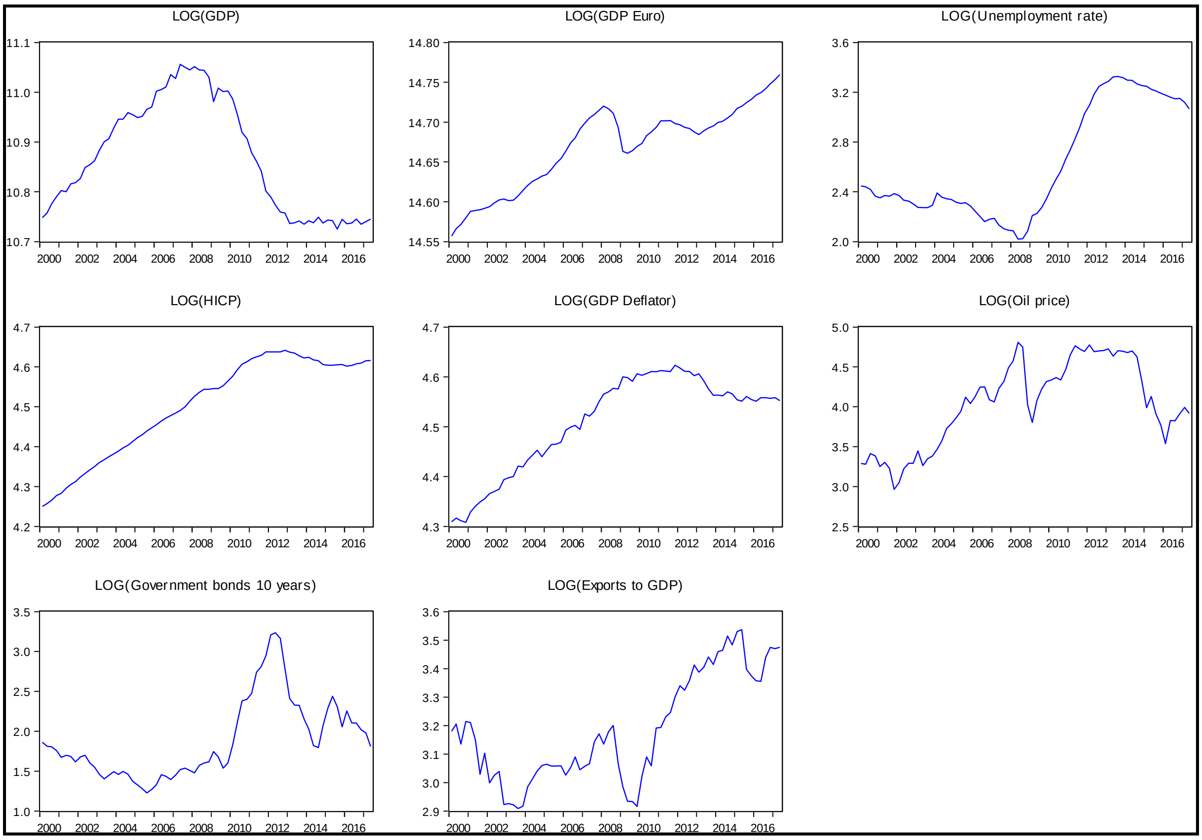
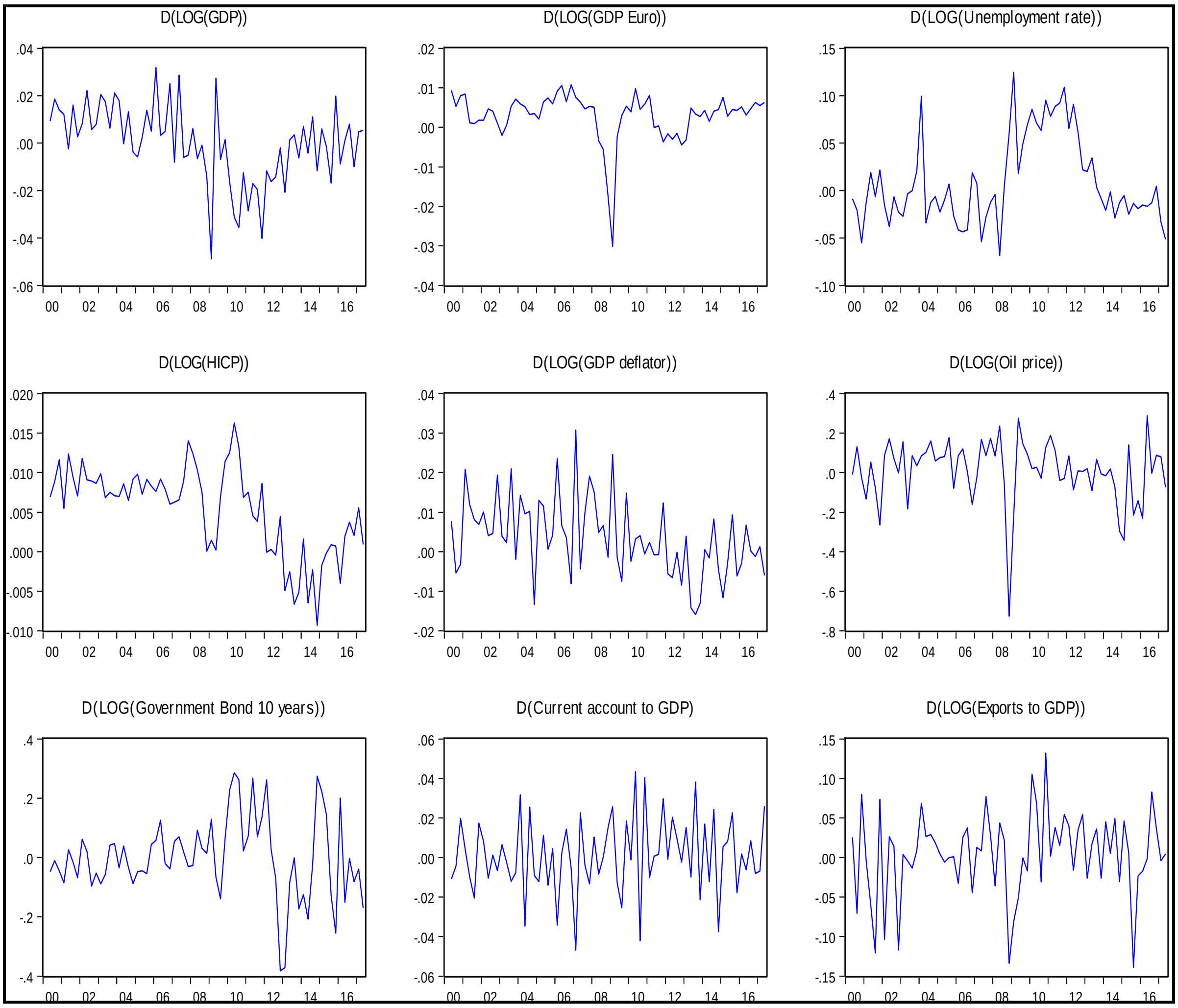


Figure 3: first difference presentation of the variables



3.2. Estimation of the model

3.2.1 Vector Autoregressive Model results

The estimation of a VAR model requires testing the stability of the series, beginning with unit root tests because, when the series under investigation are not stable, then the estimated results are not valid (spurious regression). After testing for the existence of a unit root in the series in the context of exogenous as well as endogenous breaks, we find that all variables have a unit root.

Table 2: VAR Lag Order Selection Criteria

Model 1

Endogenous variables: LOG(Y) LOG(HICP) LOG(UN) CAY

Exogenous variables: C D(LOG(OILP)) D(LOG(Y_EURO)) @TREND

Lag	LogL	LR	FPE	AIC	SC	HQ
0	471,6403	NA	1,19E-11	-13,80728	-13,27646	-13,59753
1	832,709	634,6055	3,42E-16	-24,26391	-23.20226*	-23,8444
2	862,9771	49.52954*	2.25e-16*	-24.69627*	-23,1038	-24.06701*
3	875,9481	19,65314	2,52E-16	-24,60449	-22,48119	-23,76547
4	893,2845	24,1659	2,52E-16	-24,64499	-21,99086	-23,59621

Model 2

Endogenous variables: LOG(Y) LOG(P) LOG(GB10Y) LOG(UN) LOG(XY)

Exogenous variables: C @TREND

Lag	LogL	LR	FPE	AIC	SC	HQ
0	344,1152	NA	2,76E-11	-10,1247	-9,792938	-9,993608
1	738,3555	704,8537	3,83E-16	-21,3138	-20.15262*	-20,85496
2	774,2691	58.76771*	2.80e-16*	-21.64452*	-19,65392	-20.85794*
3	788,7272	21,46818	4,00E-16	-21,32507	-18,50506	-20,21075
4	804,9116	21,57924	5,62E-16	-21,05793	-17,4085	-19,61587

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level), FPE: Final prediction error
AIC: Akaike information criterion, SC: Schwarz information criterion, HQ: Hannan-Quinn information criterion

So, we examine the short-run relationship among the series, through the estimation of alternative VAR models over the whole sample period. Specifically, we estimate VAR models using two sets of variables. First, we use as endogenous variables the real GDP, the HICP, the unemployment rate and the current account to GDP ratio. Moreover, we use the real GDP of Eurozone and the oil prices as exogenous variables. The endogenous variables, except for the current account, are in logarithms and the exogenous variables are in first differences of their logarithms. The specification of model 1 follows:

$$y_t = \mu_y + \lambda_y t + \sum_{i=1}^{2} \beta_{y,i}^{y} y_{t-i} + \sum_{i=1}^{2} \beta_{p,i}^{y} p_{t-i} + \sum_{i=1}^{2} \beta_{u,i}^{y} u_{t-i} + \sum_{i=1}^{2} \beta_{c,i}^{y} cay_{t-i} + \beta_{oil}^{y} \Delta oil_t + \beta_{ye}^{y} \Delta y_t^{euro} + \varepsilon_t^{y}$$

$$p_t = \mu_p + \lambda_p t + \sum_{i=1}^{2} \beta_{y,i}^{p} y_{t-i} + \sum_{i=1}^{2} \beta_{p,i}^{p} p_{t-i} + \sum_{i=1}^{2} \beta_{u,i}^{p} u_{t-i} + \sum_{i=1}^{2} \beta_{c,i}^{p} cay_{t-i} + \beta_{oil}^{p} \Delta oil_t + \beta_{ye}^{p} \Delta y_t^{euro} + \varepsilon_t^{p}$$

$$u_t = \mu_u + \lambda_u t + \sum_{i=1}^{2} \beta_{y,i}^{u} y_{t-i} + \sum_{i=1}^{2} \beta_{p,i}^{u} p_{t-i} + \sum_{i=1}^{2} \beta_{u,i}^{u} u_{t-i} + \sum_{i=1}^{2} \beta_{c,i}^{u} cay_{t-i} + \beta_{oil}^{u} \Delta oil_t + \beta_{ye}^{u} \Delta y_t^{euro} + \varepsilon_t^{u}$$

$$cay_t = \mu_{ca} + \lambda_{ca} t + \sum_{i=1}^{2} \beta_{y,i}^{ca} y_{t-i} + \sum_{i=1}^{2} \beta_{p,i}^{ca} p_{t-i} + \sum_{i=1}^{2} \beta_{u,i}^{ca} u_{t-i} + \sum_{i=1}^{2} \beta_{c,i}^{ca} cay_{t-i} + \beta_{oil}^{ca} \Delta oil_t + \beta_{ye}^{ca} \Delta y_t^{euro} + \varepsilon_t^{ca}$$

In the second set, we use the real GDP, the GDP deflator, the unemployment rate, the ten year government bond of Greece and the exports to GDP ratio. All variables are in logarithms. So, model 2 takes the following form:

$$y_t = \mu_y + \lambda_y t + \sum_{i=1}^{2} \beta_{y,i}^{y} y_{t-i} + \sum_{i=1}^{2} \beta_{p,i}^{y} p_{t-i} + \sum_{i=1}^{2} \beta_{u,i}^{y} u_{t-i} + \sum_{i=1}^{2} \beta_{gb,i}^{y} gb_{t-i} + \sum_{i=1}^{2} \beta_{ex,i}^{y} exy_{t-i} + \varepsilon_t^{y}$$

$$p_t = \mu_p + \lambda_p t + \sum_{i=1}^{2} \beta_{y,i}^{p} y_{t-i} + \sum_{i=1}^{2} \beta_{p,i}^{p} p_{t-i} + \sum_{i=1}^{2} \beta_{u,i}^{p} u_{t-i} + \sum_{i=1}^{2} \beta_{gb,i}^{p} gb_{t-i} + \sum_{i=1}^{2} \beta_{ex,i}^{p} exy_{t-i} + \varepsilon_t^{p}$$

$$u_t = \mu_u + \lambda_u t + \sum_{i=1}^{2} \beta_{y,i}^{u} y_{t-i} + \sum_{i=1}^{2} \beta_{p,i}^{u} p_{t-i} + \sum_{i=1}^{2} \beta_{u,i}^{u} u_{t-i} + \sum_{i=1}^{2} \beta_{gb,i}^{u} gb_{t-i} + \sum_{i=1}^{2} \beta_{ex,i}^{u} exy_{t-i} + \varepsilon_t^{u}$$

$$gb_t = \mu_{gb} + \lambda_{gb} t + \sum_{i=1}^{2} \beta_{y,i}^{gb} y_{t-i} + \sum_{i=1}^{2} \beta_{p,i}^{gb} p_{t-i} + \sum_{i=1}^{2} \beta_{u,i}^{gb} u_{t-i} + \sum_{i=1}^{2} \beta_{gb,i}^{gb} gb_{t-i} + \sum_{i=1}^{2} \beta_{ex,i}^{gb} exy_{t-i} + \varepsilon_t^{gb}$$

$$exy_t = \mu_{ex} + \lambda_{ex} t + \sum_{i=1}^{2} \beta_{y,i}^{ex} y_{t-i} + \sum_{i=1}^{2} \beta_{p,i}^{ex} p_{t-i} + \sum_{i=1}^{2} \beta_{u,i}^{ex} u_{t-i} + \sum_{i=1}^{2} \beta_{gb,i}^{ex} gb_{t-i} + \sum_{i=1}^{2} \beta_{ex,i}^{ex} exy_{t-i} + \varepsilon_t^{ex}$$

In order to test the statistical adequacy assumption, for the two sets of variables, we employ a series of misspecification tests which can be found in Table 2. In light of the tests undertaken, the VAR model includes two lags, a constant and a trend for both set of variables. The corresponding estimated VAR models are presented in tables 3.1 and 3.2.

According to the estimation results, it is obvious that our variables are connected with a short-run relationship. Tables 3.1 and 3.2 suggest that there is a strong positive relationship between variables and their first lagged value except for the current account to GDP ratio in model 1.

Table 3.1: Vector Autoregression Estimates of Model 1

	LOG(Y)	LOG(HICP)	LOG(UN)	CAY
LOG(Y(-1))	0,605208	-0,009044	-0,05987	-0,076807
	[4.91838]	[-0.27740]	[-0.18299]	[-0.48230]
LOG(Y(-2))	0,326606	0,02391	-0,408086	0,136118
	[2.50233]	[0.69138]	[-1.17590]	[0.80582]
LOG(HICP(-1))	-1,17358	1,232208	0,847003	0,096972
	[-2.79091]	[11.0595]	[0.75756]	[0.17819]
LOG(HICP(-2))	1,215173	-0,30795	-0,084698	-0,446336
	[2.95057]	[-2.82206]	[-0.07735]	[-0.83740]
LOG(UN(-1))	-0,158965	0,02381	1,39361	0,082617
	[-3.06139]	[1.73056]	[10.0938]	[1.22939]
LOG(UN(-2))	0,109049	-0,016726	-0,552377	0,02823
	[2.36636]	[-1.36983]	[-4.50810]	[0.47333]
CAY(-1)	0,036081	-0,057348	-0,103511	0,02596
	[0.36055]	[-2.16286]	[-0.38902]	[0.20044]
CAY(-2)	0,186523	-0,034797	0,181913	0,189718
	[1.82187]	[-1.28278]	[0.66826]	[1.43184]
C	0,717113	0,145868	2,171612	0,537802
	[1.26430]	[0.97060]	[1.43993]	[0.73263]
D(LOG(OILP))	0,012169	0,004824	-0,055458	-0,007592
	[1.26628]	[1.89445]	[-2.17030]	[-0.61038]
D(LOG(Y_EURO))	1,004911	0,124158	-0,043749	-0,843982
	[3.35101]	[1.56258]	[-0.05487]	[-2.17461]
@TREND	-0,000301	0,00031	-0,002576	0,001175
	[-0.70024]	[2.72515]	[-2.25648]	[2.11554]
R-squared	0,992117	0,999486	0,996495	0,923763
Adj. R-squared	0,990568	0,999385	0,995806	0,908788
Log likelihood	888,0574			
Akaike information criterion	-24,70757			
Schwarz criterion	-23,14086			

t-statistics in [ ]

Table 3.2: Vector Autoregression Estimates of Model 2					
	LOG(Y)	**LOG(P)**	**LOG(GB10Y)**	**LOG(UN)**	**LOG(XY)**
LOG(Y(-1))	0,697925	-0,000717	0,575414	-0,329393	0,615996
	[4.77539]	[-0.00763]	[0.42115]	[-0.92761]	[1.02575]
LOG(Y(-2))	0,287891	0,161948	-1,68069	-0,168983	-0,276765
	[1.94743]	[1.70386]	[-1.21612]	[-0.47046]	[-0.45562]
LOG(P(-1))	0,359986	0,465311	2,069688	-0,479129	0,156621
	[1.77963]	[3.57778]	[1.09447]	[-0.97487]	[0.18843]
LOG(P(-2))	-0,403488	0,266761	-0,372432	1,095765	-0,162081
	[-2.19786]	[2.26005]	[-0.21701]	[2.45662]	[-0.21486]
LOG(GB10Y(-1))	-0,016466	-0,005663	1,252973	0,036951	0,085746
	[-1.30043]	[-0.69567]	[10.5854]	[1.20113]	[1.64812]
LOG(GB10Y(-2))	0,005111	0,01615	-0,524833	-0,021722	-0,060679
	[0.39145]	[1.92387]	[-4.29995]	[-0.68475]	[-1.13106]
LOG(UN(-1))	-0,131473	0,066078	1,325927	1,159101	-0,159131
	[-2.19561]	[1.71634]	[2.36861]	[7.96692]	[-0.64675]
LOG(UN(-2))	0,156524	-0,063848	-1,448717	-0,287267	0,350436
	[2.71085]	[-1.71989]	[-2.68388]	[-2.04767]	[1.47705]
LOG(XY(-1))	-0,058653	0,000834	0,477442	-0,038431	0,679811
	[-1.84307]	[0.04078]	[1.60483]	[-0.49703]	[5.19879]
LOG(XY(-2))	0,021048	0,00721	-0,239571	-0,047597	0,0277
	[0.67572]	[0.36001]	[-0.82270]	[-0.62890]	[0.21641]
C	0,430025	-0,622466	4,602551	3,261287	-3,269104
	[0.67876]	[-1.52814]	[0.77710]	[2.11866]	[-1.25578]
@TREND	-0,000114	0,000868	-0,005128	-0,000959	-0,000147
	[-0.28663]	[3.39527]	[-1.37952]	[-0.99305]	[-0.09013]
R-squared	0,990731	0,994072	0,958993	0,996559	0,944671
Adj. R-squared	0,98891	0,992908	0,950939	0,995883	0,933803
Log likelihood	793,1515				
AIC	-21,56328				
Schwarz criterion	-19,60489				

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3.2.2 Granger Causality Analysis

Our estimation results provide evidence which supports the existence of a short run relationship among the variables. In order to verify this correlation we perform Granger Causality Tests, which are presented in Tables 4.1 and 4.2 for each model correspondingly. Particularly, we test the null hypothesis that there is no Granger Causality relationship in the system, for the above two VAR models. For each equation in the VAR models, the tables display (Wald) statistics for the joint significance of each and of all other lagged endogenous variables in that equation. Consequently, the results obtained from the VAR models, are confirmed as well in the Granger Causality analysis.

Table 4.1: Pairwise Granger Causality Tests-Block Exogeneity Wald Tests

Dependent variable: LOG(Y)			
Excluded	Chi-sq	df	Prob.
LOG(HICP)	8,775928	2	0,0124
LOG(UN)	9,49817	2	0,0087
CAY	3,611581	2	0,1643
All	38,75431	6	0
Dependent variable: LOG(HICP)			
Excluded	Chi-sq	df	Prob.
LOG(Y)	0,821575	2	0,6631
LOG(UN)	3,014998	2	0,2215
CAY	6,930216	2	0,0313
All	23,33428	6	0,0007
Dependent variable: LOG(UN)			
Excluded	Chi-sq	df	Prob.
LOG(Y)	6,384426	2	0,0411
LOG(HICP)	7,473642	2	0,0238
CAY	0,55248	2	0,7586
All	13,15319	6	0,0407
Dependent variable: CAY			
Excluded	Chi-sq	df	Prob.
LOG(Y)	0,795416	2	0,6719
LOG(HICP)	7,114945	2	0,0285
LOG(UN)	9,022773	2	0,011
All	27,18595	6	0,0001

Table 4.2: Pairwise Granger Causality Tests-Block Exogeneity Wald Tests

Dependent variable: LOG(Y)			
Excluded	Chi-sq	df	Prob.
LOG(P)	4,906441	2	0,086
LOG(GB10Y)	2,777799	2	0,2493
LOG(UN)	8,710586	2	0,0128
LOG(XY)	4,700071	2	0,0954
All	41,05808	8	0
Dependent variable: LOG(P)			
Excluded	Chi-sq	df	Prob.
LOG(Y)	10,39165	2	0,0055
LOG(GB10Y)	5,528191	2	0,063
LOG(UN)	3,031553	2	0,2196
LOG(XY)	0,371078	2	0,8307
All	33,01018	8	0,0001
Dependent variable: LOG(GB10Y)			
Excluded	Chi-sq	df	Prob.
LOG(Y)	2,83311	2	0,2425
LOG(P)	3,448951	2	0,1783
LOG(UN)	7,501151	2	0,0235
LOG(XY)	2,972298	2	0,2262
All	17,3361	8	0,0268
Dependent variable: LOG(UN)			
Excluded	Chi-sq	df	Prob.
LOG(Y)	6,578263	2	0,0373
LOG(P)	11,67087	2	0,0029
LOG(GB10Y)	1,631372	2	0,4423
LOG(XY)	2,705687	2	0,2585
All	27,73622	8	0,0005
Dependent variable: LOG(XY)			
Excluded	Chi-sq	df	Prob.
LOG(Y)	1,67758	2	0,4322
LOG(P)	0,046173	2	0,9772
LOG(GB10Y)	2,796295	2	0,2471
LOG(UN)	7,728496	2	0,021
All	14,47257	8	0,0702

3.2.3 Cointegration Analysis

Although the VAR results provide information about the short-run relationship between the macroeconomic variables, nevertheless we do not know what their long-run behaviour is. The VECM not only gives an answer to the question of whether the short-run relationship of the variables is persistent, but also allows us to perform forecasting.

The estimation of the VECM requires first to test for the existence of cointegration. We follow the Johansen and Juselius (1990, 1992) approach which is based on canonical correlations. As we determine that the number of lags is two in the above VAR models then we should impose actually one lag in the VECM, in the cointegration test. The results are presented in Tables 5.1 and 5.2 for each model respectively.

Table 5.1: Johansen Cointegration Test for Model 1

Trend assumption: Linear deterministic trend (restricted)
Series: LOG(Y) LOG(HICP) LOG(UN) CAY
Exogenous series: D(LOG(OILP)) D(LOG(Y_EURO))
Warning: Critical values assume no exogenous series
Lags interval (in first differences): 1 to 1

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0,604983	119,4953	63,8761	0
At most 1 *	0,410417	56,33511	42,91525	0,0014
At most 2	0,16465	20,40796	25,87211	0,206
At most 3	0,113268	8,174421	12,51798	0,2378

Trace test indicates 2 cointegrating eqn(s) at the 0.05 level

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0,604983	63,16021	32,11832	0
At most 1 *	0,410417	35,92715	25,82321	0,0017
At most 2	0,16465	12,23354	19,38704	0,3937
At most 3	0,113268	8,174421	12,51798	0,2378

Max-eigenvalue test indicates 2 cointegrating eqn(s) at the 0.05 level

* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Table 5.2: Johansen Cointegration Test for Model 2

Trend assumption: Linear deterministic trend (restricted)
Series: LOG(Y) LOG(P) LOG(GB10Y) LOG(UN) LOG(XY)
Lags interval (in first differences): 1 to 1

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0,05 Critical Value	Prob.**
None *	0,530589	136,6277	88,8038	0
At most 1 *	0,37613	85,20094	63,8761	0,0003
At most 2 *	0,301252	53,11769	42,91525	0,0036
At most 3 *	0,245541	28,74202	25,87211	0,0214
At most 4	0,131444	9,582758	12,51798	0,1474

Trace test indicates 4 cointegrating eqn(s) at the 0.05 level

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0,05 Critical Value	Prob.**
None *	0,530589	51,42675	38,33101	0,001
At most 1	0,37613	32,08325	32,11832	0,0505
At most 2	0,301252	24,37567	25,82321	0,0767
At most 3	0,245541	19,15926	19,38704	0,0539
At most 4	0,131444	9,582758	12,51798	0,1474

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Table 5.1 suggests that, taking into account the Trace Statistic and the Maximal Eigenvalue Statistic, we identify the existence of two cointegrating relationships in the four-variable VAR with two exogenous variables at the 5%. Regarding Table 5.2, the Trace Statistic indicates the existence of four cointegrating relationships while the Maximal Eigenvalue Statistic of one cointegrating equation. Taking into consideration the Maximal Eigenvalue Statistic we proceed with one cointegrating equation at the 5% in the five variable VAR.

As a result, since both models exhibit two and one cointegrating relationships between the variables respectively, we move a step further for the estimation of two VEC models which require not only the variables to be linked in the short run, but to be related in the long run via the existence of cointegration.

3.2.4 Vector Error Correction Estimation

In this section we estimate a VECM model based on the four-variable VAR model with two exogenous variables in which we identify two cointegrating relationships. The specification of the first model follows:

$$\begin{aligned}
\Delta y_t = \mu_1 + \alpha_{11}\left(c_1 + c_2 t + c_3 y_{t-1} + c_4 p_{t-1} + c_5 u_{t-1} + c_6 cay_{t-1}\right) + \alpha_{12}\left(d_1 + d_2 t + d_3 y_{t-1} + d_4 p_{t-1} + d_5 u_{t-1} + d_6 cay_{t-1}\right) + \\
+\beta_{11}\Delta y_{t-1} + \beta_{12}\Delta p_{t-1} + \beta_{13}\Delta u_{t-1} + \beta_{14}\Delta cay_{t-1} + \beta_{15}\Delta oil_t + \beta_{16}\Delta y_t^{euro} + \varepsilon_t^{y}
\end{aligned}$$

$$\begin{aligned}
\Delta p_t = \mu_2 + \alpha_{21}\left(c_1 + c_2 t + c_3 y_{t-1} + c_4 p_{t-1} + c_5 u_{t-1} + c_6 cay_{t-1}\right) + \alpha_{22}\left(d_1 + d_2 t + d_3 y_{t-1} + d_4 p_{t-1} + d_5 u_{t-1} + d_6 cay_{t-1}\right) + \\
+\beta_{21}\Delta y_{t-1} + \beta_{22}\Delta p_{t-1} + \beta_{23}\Delta u_{t-1} + \beta_{24}\Delta cay_{t-1} + \beta_{25}\Delta oil_t + \beta_{26}\Delta y_t^{euro} + \varepsilon_t^{p}
\end{aligned}$$

$$\begin{aligned}
\Delta u_t = \mu_3 + \alpha_{31}\left(c_1 + c_2 t + c_3 y_{t-1} + c_4 p_{t-1} + c_5 u_{t-1} + c_6 cay_{t-1}\right) + \alpha_{32}\left(d_1 + d_2 t + d_3 y_{t-1} + d_4 p_{t-1} + d_5 u_{t-1} + d_6 cay_{t-1}\right) + \\
+\beta_{31}\Delta y_{t-1} + \beta_{32}\Delta p_{t-1} + \beta_{33}\Delta u_{t-1} + \beta_{34}\Delta cay_{t-1} + \beta_{35}\Delta oil_t + \beta_{36}\Delta y_t^{euro} + \varepsilon_t^{u}
\end{aligned}$$

$$\begin{aligned}
\Delta cay_t = \mu_4 + \alpha_{41}\left(c_1 + c_2 t + c_3 y_{t-1} + c_4 p_{t-1} + c_5 u_{t-1} + c_6 cay_{t-1}\right) + \alpha_{42}\left(d_1 + d_2 t + d_3 y_{t-1} + d_4 p_{t-1} + d_5 u_{t-1} + d_6 cay_{t-1}\right) + \\
+\beta_{41}\Delta y_{t-1} + \beta_{42}\Delta p_{t-1} + \beta_{43}\Delta u_{t-1} + \beta_{44}\Delta cay_{t-1} + \beta_{45}\Delta oil_t + \beta_{46}\Delta y_t^{euro} + \varepsilon_t^{ca}
\end{aligned}$$

The VECM results are presented in Table 6.1. The two cointegrated equations summarize the long run behavior of the variables. The unemployment rate is related negatively with real GDP and HICP while the current account to GDP ratio is related positively with real GDP and negatively with HICP.

Table 6.1: Vector Error Correction Estimates of Model 1

Cointegrating Eq	CointEq1	CointEq2
LOG(Y(-1))	1	0
LOG(HICP(-1))	0	1
LOG(UN(-1))	0.784695	0.036816
	[6.78376]	[0.85099]
CAY(-1)	-1.662007	0.727937
	[-1.93641]	[2.26765]
@TREND(00Q1)	9.00E-05	-0.003584
	[0.06183]	[-6.58113]
C	-13.05792	-4.416736

Error Correction	D(LOG(Y))	D(LOG(HICP))	D(LOG(UN))	D(CAY)
CointEq1	-0.072544	0.013726	-0.135727	0.192264
	[-2.10719]	[1.51143]	[-1.42668]	[4.32843]
CointEq2	0.043293	-0.074053	0.452494	-0.47601
	[0.44862]	[-2.90892]	[1.69681]	[-3.82304]
D(LOG(Y(-1)))	-0.3	-0.029162	0.146206	-0.214802
	[-2.42101]	[-0.89212]	[0.42697]	[-1.34351]
D(LOG(HICP(-1)))	-1.544092	0.385542	1.491248	0.659268
	[-4.09692]	[3.87778]	[1.43183]	[1.35574]
D(LOG(UN(-1)))	-0.120432	0.018839	0.683838	0.013492
	[-2.89020]	[1.71385]	[5.93876]	[0.25095]
D(CAY(-1))	-0.132198	0.022421	-0.477602	-0.251188
	[-1.35077]	[0.86842]	[-1.76594]	[-1.98922]
C	0.005877	0.002719	-0.001693	0.000748
	[2.43946]	[4.27881]	[-0.25438]	[0.24056]
D(LOG(OILP))	0.010899	0.005182	-0.058454	-0.010268
	[1.12672]	[2.03082]	[-2.18684]	[-0.82271]
D(LOG(Y_EURO))	1.182096	0.084	-1.038706	-1.057143
	[4.23101]	[1.13972]	[-1.34537]	[-2.93262]
R-squared	0.582659	0.747873	0.596446	0.501241
Adj. R-squared	0.52607	0.713686	0.541727	0.433613
Log likelihood		877.8534		
AIC		-24.46628		
Schwarz criterion		-22.96485		

t-statistics in [ ]

Then we estimate a VECM model based on the five-variable VAR model in which we identify one cointegrating relationship. The VECM for model 2 follows:

$$\begin{aligned}\Delta y_t = {} & \mu_1 + \alpha_1 \left(c_1 + c_2 t + c_3 y_{t-1} + c_4 p_{t-1} + c_5 u_{t-1} + c_6 gb_{t-1} + c_7 exy_{t-1} \right) + \\ & + \beta_{11} \Delta y_{t-1} + \beta_{12} \Delta p_{t-1} + \beta_{13} \Delta u_{t-1} + \beta_{14} \Delta gb_{t-1} + \beta_{15} \Delta exy_{t-1} + \varepsilon_t^y\end{aligned}$$

$$\begin{aligned}\Delta p_t = {} & \mu_2 + \alpha_2 \left(c_1 + c_2 t + c_3 y_{t-1} + c_4 p_{t-1} + c_5 u_{t-1} + c_6 gb_{t-1} + c_7 exy_{t-1} \right) + \\ & + \beta_{21} \Delta y_{t-1} + \beta_{22} \Delta p_{t-1} + \beta_{23} \Delta u_{t-1} + \beta_{24} \Delta gb_{t-1} + \beta_{25} \Delta exy_{t-1} + \varepsilon_t^p\end{aligned}$$

$$\begin{aligned}\Delta u_t = {} & \mu_3 + \alpha_3 \left(c_1 + c_2 t + c_3 y_{t-1} + c_4 p_{t-1} + c_5 u_{t-1} + c_6 gb_{t-1} + c_7 exy_{t-1} \right) + \\ & + \beta_{31} \Delta y_{t-1} + \beta_{32} \Delta p_{t-1} + \beta_{33} \Delta u_{t-1} + \beta_{34} \Delta gb_{t-1} + \beta_{35} \Delta exy_{t-1} + \varepsilon_t^u\end{aligned}$$

$$\begin{aligned}\Delta gb_t = {} & \mu_4 + \alpha_4 \left(c_1 + c_2 t + c_3 y_{t-1} + c_4 p_{t-1} + c_5 u_{t-1} + c_6 gb_{t-1} + c_7 exy_{t-1} \right) + \\ & + \beta_{41} \Delta y_{t-1} + \beta_{42} \Delta p_{t-1} + \beta_{43} \Delta u_{t-1} + \beta_{44} \Delta gb_{t-1} + \beta_{45} \Delta exy_{t-1} + \varepsilon_t^{gb}\end{aligned}$$

$$\begin{aligned}\Delta exy_t = {} & \mu_5 + \alpha_5 \left(c_1 + c_2 t + c_3 y_{t-1} + c_4 p_{t-1} + c_5 u_{t-1} + c_6 gb_{t-1} + c_7 exy_{t-1} \right) + \\ & + \beta_{51} \Delta y_{t-1} + \beta_{52} \Delta p_{t-1} + \beta_{53} \Delta u_{t-1} + \beta_{54} \Delta gb_{t-1} + \beta_{55} \Delta exy_{t-1} + \varepsilon_t^{exy}\end{aligned}$$

The VECM results are presented in Table 6.2. The one cointegrated equation indicates that the deflator is related positively with real GDP while the unemployment rate, the ten-year government bond and the exports to GDP ratio are related negatively with real GDP.

Table 6.2: Vector Error Correction Estimates of Model 2

Cointegrating Eq	CointEq1
LOG(Y(-1))	1
LOG(P(-1))	-1,778726
	[-8.62352]
LOG(GB10Y(-1))	0,015208
	[0.60397]
LOG(UN(-1))	0,070384
	[1.63886]
LOG(XY(-1))	0,098503
	[1.27477]
@TREND(00Q1)	0,004907
	[3.75484]
C	-3,548537

Error Correction	D(LOG(Y))	D(LOG(P))	D(LOG(GB10Y))	D(LOG(UN))	D(LOG(XY))
CointEq1	0,058269	0,138054	0,156525	-0,154519	-0,29327
	[1.58268]	[6.26711]	[0.46263]	[-1.68071]	[-1.96214]
D(LOG(Y(-1)))	-0,147014	-0,166094	1,093608	0,209993	1,013592
	[-0.96870]	[-1.82916]	[0.78414]	[0.55411]	[1.64514]
D(LOG(P(-1)))	0,415917	-0,231596	1,75416	-0,461335	0,005733
	[2.26557]	[-2.10846]	[1.03977]	[-1.00633]	[0.00769]
D(LOG(GB10Y(-1)))	-0,018992	-0,007706	0,40438	0,055266	0,076764
	[-1.45237]	[-0.98491]	[3.36509]	[1.69249]	[1.44602]
D(LOG(UN(-1)))	-0,12005	0,091752	0,950694	0,532172	-0,012826
	[-2.33338]	[2.98060]	[2.01076]	[4.14220]	[-0.06141]
D(LOG(XY(-1)))	-0,048192	-0,006526	0,237216	-0,019486	-0,054015
	[-1.56514]	[-0.35423]	[0.83834]	[-0.25343]	[-0.43211]
C	-0,000303	0,003433	-0,017445	0,005684	0,004289
	[-0.16967]	[3.20845]	[-1.06142]	[1.27271]	[0.59075]
R-squared	0,409783	0,419242	0,280693	0,53396	0,098995
Adj. R-squared	0,351729	0,362118	0,209942	0,48812	0,010372
Log likelihood		750,551			
AIC		-20,86915			
Schwarz criterion		-19,53091			

t-statistics in [ ]

3.2.5 Variance Decomposition Analysis

Using the estimated models, which provide information for the long-run relationship of the variables, we perform Variance Decomposition Analysis which is a way to characterize the dynamic behavior of the models. Table 7.1 suggests that in the long run, the variation of real GDP depends also on shocks to other variables. Specifically, this percentage increases through time and, in the last period, about 42% of the total change on the variance is due to the rest variables. A similar situation holds for the rest variables with a notable impact on current account to GDP ratio.

Table 7.1: Variance Decomposition Analysis of Model 1

Period	**Variance Decomposition of:** depending on:	**LOG(Y)** LOG(Y)	**LOG(HICP)** LOG(HICP)	**LOG(UN)** LOG(UN)	**CAY** CAY
1		100.00	80.59	80.60	86.62
2		86.41	72.18	77.44	76.94
3		78.34	67.07	73.82	61.76
4		71.59	63.58	70.45	50.19
5		67.50	61.81	67.97	42.00
6		64.37	61.07	66.21	36.19
7		62.13	60.90	64.99	31.99
8		60.33	60.95	64.09	28.62
9		58.90	60.99	63.41	25.94
10		57.72	60.83	62.87	23.74

The dynamic behavior of the second model is similar to that of the first. More specifically, Table 7.2 indicates that the impact on variance decomposition of the GDP deflator from other variables is very strong. Through time, the influence increases and in the last period, 52% of the variation of GDP deflator is due to the other variables. Regarding the unemployment rate, the impact on its variation from the rest variables increases reaching a level of 39% in the last period. Finally, the variation of the rest three variables, namely the real GDP, the ten-year government bond and the exports to GDP ratio, depends also on shocks to other variables on average 15%-20% during the last period.

Consequently, in the long run, the link between the variables becomes more significant, since the variation of a variable is due not only to own, but to shocks from other variables too.

Table 7.2: Variance Decomposition Analysis of Model 2

Period	Variance Decomposition of: depending on:	LOG(Y) LOG(Y)	LOG(P) LOG(P)	LOG(GB10Y) LOG(GB10Y)	LOG(UN) LOG(UN)	LOG(XY) LOG(XY)
1		100,00	86,65	93,16	80,90	97,88
2		93,20	73,52	90,77	79,78	93,36
3		89,74	69,60	87,97	76,11	92,32
4		87,40	67,78	85,76	72,82	92,00
5		85,35	65,83	84,14	70,07	91,48
6		83,71	63,64	82,97	67,67	90,86
7		82,42	60,97	82,12	65,55	90,13
8		81,38	57,56	81,49	63,66	89,30
9		80,52	53,38	81,03	61,96	88,35
10		79,80	48,61	80,69	60,40	87,32

3.2.6 Forecasting Performance

The VECMs are used to produce medium-term forecasts for main macroeconomic variables. According to the estimated models, we make forecasts for the endogenous variables for the next two years (eight quarters). Regarding the first model, we need to obtain forecasted values for the two exogenous variables, namely the oil prices and the real GDP of Eurozone. For this reason, we examine alternative univariate autoregressive models for each one of the two variables and choose the model with the minimum root mean squared error. So, for oil price we estimate an AR(3) specification while for the real GDP of Eurozone an AR(2) model. Then, we may estimate their eight-quarter ahead forecasts and use them in order to estimate the forecasted values of the endogenous variables.

The estimated forecasts of the endogenous variables are presented in Tables 8.1 and 8.2 respectively. These tables display the average of the growth rate of the seasonally adjusted real GDP, the growth rate of the HICP, the growth rate of the GDP deflator, the unemployment rate, the current account to GDP ratio and the exports to GDP ratio. All values are annually averages.

In a second stage, following Anderson et al (2002), we assess the forecasting performance of the estimated VECMs. We estimate each model during the sample period 2000:1 to 2014:4 and make forecasts for the next eight quarters. Then we compare the forecasted values with actual data for the periods 2015:1 to 2016:4 and compute the corresponding RMSE criterion. These results are presented in the last raw of each table. We may see that model 2 performs better in terms of real GDP.

Table 8.1: Forecasts of Model 1

		Real GDP	HICP	Unemployment rate	Current account to GDP ratio
2017	Q_1^* (y-o-y)	0,39%	1,34%	22,60%	-2,33%
	Q_2^*(y-o-y)	0,82%	1,23%	21,47%	0,28%
	Q_3(y-o-y)	0,76%	0,81%	20,70%	-2,14%
	Q_4(y-o-y)	2,84%	0,61%	20,36%	-1,88%
	Average	**1,20%**	**1,00%**	**21,28%**	**-1,52%**
2018	Q_1^* (y-o-y)	2,62%	0,17%	20,07%	-2,27%
	Q_2^*(y-o-y)	2,41%	0,29%	19,85%	-1,98%
	Q_3(y-o-y)	1,75%	0,59%	19,67%	-1,99%
	Q_4(y-o-y)	0,83%	0,85%	19,56%	-1,87%
	Average	**1,90%**	**0,47%**	**19,79%**	**-2,03%**
RMSE		0,0144	0,0056	0,0278	0,0132

Note: RMSE stands for Mean Squared Error
* denotes realized values
y-o-y: with respect to the corresponding quarter of the previous year

Table 8.2: Forecasts of Model 2

		Real GDP	GDP deflator	Government bond 10y	Unemployment rate	Exports to GDP ratio
2017	Q_1^* (y-o-y)	0,39%	0,70%	7,24%	22,60%	32,14%
	Q_2^*(y-o-y)	0,82%	-0,57%	6,11%	21,47%	32,29%
	Q_3(y-o-y)	0,75%	-0,15%	5,34%	20,80%	31,93%
	Q_4(y-o-y)	2,67%	0,29%	4,90%	20,32%	31,75%
	Average	**1,16%**	**0,07%**	**5,90%**	**21,30%**	**32,03%**
2018	Q_1^* (y-o-y)	2,82%	0,58%	4,64%	20,00%	31,69%
	Q_2^*(y-o-y)	2,83%	1,70%	4,48%	19,78%	31,59%
	Q_3(y-o-y)	2,61%	1,78%	4,38%	19,61%	31,52%
	Q_4(y-o-y)	2,17%	2,00%	4,31%	19,48%	31,46%
	Average	**2,61%**	**1,51%**	**4,45%**	**19,72%**	**31,56%**
RMSE		0,0116	0,0097	0,16	0,0356	0,0595

Note: RMSE stands for Mean Squared Error
* denotes realized values
y-o-y: with respect to the corresponding quarter of the previous year

4. Conclusion

This study has performed a forecasting exercise involving two time series datasets for Greece. Due to the identification of cointegrating relationships in the variables, short-term forecasts of GDP are estimated applying Johansen's VECM estimation method using an information set that proxies for the components of expenditure based GDP within an open economy framework. For this purpose, the models are estimated using quarterly data on real GDP, the GDP price deflator, HICP, unemployment rate, 10yr government bond rates, exports to GDP ratio and the current account to GDP ratio over the sample period 2000:1 to 2017:2. Then six quarters out of sample forecasts are generated under each model framework. Moreover, we assess the forecasting performance of the estimated VECMs estimating each model during the sample period 2000:1 to 2014:4, making forecasts for the next eight quarters and comparing the forecasted values with actual data. In addition to the forecasts, an effort is made to examine the relationships among the variables.

Developing this research further could take into account the fact that the models presented here are linear by their nature, and therefore fail to take into account nonlinearities in the data. One of the responses to this problem within the literature has been the development of DSGE models, which are capable of handling both structural changes, as well as nonlinearities. The current trend in forecasting is dominated by the use of calibrated and estimated versions of DSGE models that have been shown to produce better forecasts relative to traditional forecasting methods in many cases (see, e.g, Zimmerman (2001)). Another potential area to further develop the work presented here, could be to pool together the information set into a panel of European countries. Within a panel VECM framework, the predictive ability of a candidate variable within the information set could be explored for the entire panel of countries. Analysis such as this may reveal potential interdependencies within the European group of countries.

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